5 A list data structure can be represented using a fixed-size array.

The pseudo-code algorithm in **Figure 1** can be used to carry out one useful operation on a list.

Figure 1

```
p ← 1
IF ListLength > 0 THEN
   WHILE p <= ListLength AND List[p] < New
      p ← p + 1
   ENDWHILE
   FOR q ← ListLength DOWNTO p DO
      List[q + 1] ← List[q]
   ENDFOR
ENDIF
List[p] ← New
ListLength ← ListLength + 1
```

The `DOWNTO` command causes the loop counter value to decrease by one on each iteration of the `FOR` loop.

The initial values of the variables for one particular execution of the algorithm are shown in the trace table opposite, labelled **Table 1**. Array indexing starts at 1.

(a) Complete the trace table for the execution of the algorithm in **Figure 1**.

Table 1

				List				
ListLength	New	p	q	[1]	[2]	[3]	[4]	[5]
4	48	–	–	19	43	68	107	–

(4)

(b) Describe the purpose of the algorithm in **Figure 1**.

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(1)

(c) A list may be implemented using a static data structure such as a fixed-length array, or using a dynamic data structure such as a linked list.

If the list were to be implemented using a dynamic data structure explain what the heap would be used for.

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(1)
(Total 6 marks)

6 A Turing machine has been designed to recognise palindromic binary numbers, ie numbers such as `101` and `0110` that read the same from left to right as from right to left.

The machine has states S_B, S_0, S_1, S_{C0}, S_{C1}, S_L, S_Y and S_N.

S_B is the start state and S_Y and S_N are the stop states.

The machine stores data on a single tape which is infinitely long in one direction. The machine's alphabet is 0, 1 and □, where □ is the symbol used to indicate a blank cell on the tape. The machine will enter state S_Y if the value represented on the tape is a palindromic binary number, otherwise it will enter state S_N.

The transition rules for this Turing machine can be expressed as a transition function δ.

Rules are written in the form:

$$\delta(\text{Current State, Input Symbol}) = (\text{Next State, Output Symbol, Movement})$$

So, for example, the rule:

$$\delta(S_B, 0) = (S_0 , \square, \longrightarrow)$$

means:

IF the machine is currently in state S_B AND the input symbol read from the tape is 0

THEN the machine should change to state S_0, write a blank symbol (□) to the tape and move the read/write head one cell to the right

The machine's transition function, δ, is defined by:

δ $(S_B, 0) = (S_0, \square, \longrightarrow)$

δ $(S_B, 1) = (S_1, \square, \longrightarrow)$

δ $(S_B, \square) = (S_Y, \square \longrightarrow)$

δ $(S_0, 0) = (S_0, 0, \longrightarrow)$

δ $(S_0, 1) = (S_0, 1, \longrightarrow)$

δ $(S_0, \square) = (S_{C0}, \square, \longleftarrow)$

δ $(S_1, 0) = (S_1, 0, \longrightarrow)$

δ $(S_1, 1) = (S_1, 1, \longrightarrow)$

δ $(S_1, \square) = (S_{C1}, \square, \longleftarrow)$

δ $(S_{C0}, 0) = (S_L, \square, \longleftarrow)$

δ $(S_{C0}, 1) = (S_N, 1, \longleftarrow)$

δ $(S_{C0}, \square) = (S_Y, \square, \longrightarrow)$

δ $(S_{C1}, 0) = (S_N, 0, \longleftarrow)$

δ $(S_{C1}, 1) = (S_L, \square, \longleftarrow)$

δ $(S_{C1}, \square) = (S_Y, \square, \longrightarrow)$

δ $(S_L, 0) = (S_L, 0, \longleftarrow)$

δ $(S_L, 1) = (S_L, 1, \longleftarrow)$

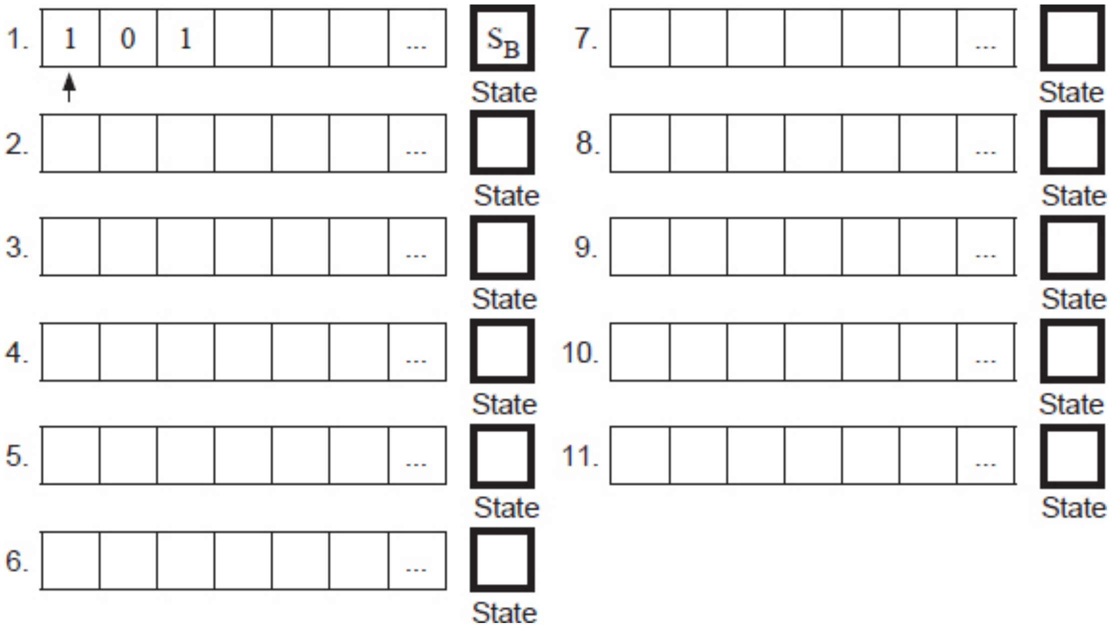
δ $(S_L, \square) = (S_B, \square, \longrightarrow)$

(a) This Turing machine is carrying out a computation. The machine starts in state S_B with the string `101` on the tape. All other cells contain the blank symbol, □ (not shown). The read/write head is located at the left hand symbol of the string and is indicated with an upward arrow.

Trace the computation of the Turing machine, using the transition function δ.

Show the contents of the tape, the current position of the read/write head and the current state as the input symbols are processed.

The initial configuration of the machine has been completed for you in step 1.



(5)

(b) The three rules shown below are part of the machine's transition function.

$$\delta\ (S_0, 0) = (S_0, 0, \longrightarrow)$$

$$\delta\ (S_0, 1) = (S_0, 1, \longrightarrow)$$

$$\delta\ (S_0, \square) = (S_{C0}, \square, \longleftarrow)$$

Explain what effect these three rules, taken together, have on the tape, the read/write head and the state of the Turing machine.

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(2)
(Total 7 marks)

7

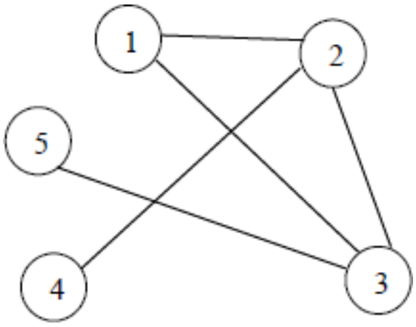
The Cat transportation company (CTC) is a business that specialises in preparing cats for cat shows.

They need to take five cats to the AQA cat show. They will transport the cats in their van. CTC owns only one van.

They cannot put all the cats in their van at the same time because some of the cats get stressed when in the company of some of the other cats. The cats would not therefore arrive in top condition for the cat show if they were all in the van at the same time.

The graph in **Figure 1** shows the relationships between the five cats (labelled 1 to 5). If there is an edge between two cats in the graph then they **cannot** travel in the van together at the same time.

Figure 1



(a) Explain why the graph in **Figure 1** is **not** a tree.

(1)

(b) Represent the graph shown in **Figure 1** as an adjacency list by completing **Table 1** .

Table 1

Vertex (in Figure 1)	**Adjacent vertices**
1	
2	
3	
4	
5	

(2)

(c) **Table 2** shows how the graph in **Figure 1** can be represented as an adjacency matrix.

Table 2

Vertex (in Figure 1)	**1**	**2**	**3**	**4**	**5**
1	0	1	1	0	0
2	1	0	1	1	0
3	1	1	0	0	1
4	0	1	0	0	0
5	0	0	1	0	0

Explain the circumstances in which it is more appropriate to represent a graph using an adjacency list instead of an adjacency matrix.

(2)

(d) **Figure 2** shows an algorithm, written in pseudo-code, that CTC use.

Figure 2

```
NoOfCats ← 5
Cat[1] ← 1
FOR A ← 2 TO NoOfCats
  B ← 1
  C ← 1
  WHILE B < A DO
    IF M[A, B] = 1
      THEN
        IF Cat[B] = C
          THEN
            B ← 1
            C ← C + 1
          ELSE B ← B + 1
        ENDIF
      ELSE B ← B + 1
    ENDIF
  ENDWHILE
  Cat[A] ← C
ENDFOR
```

The two-dimensional array, `M`, is used to store the adjacency matrix shown in **Table 2**.

Complete **Table 3** to show the result of tracing the algorithm in **Figure 2**.

Table 3

				Cat				
NoOfCats	**A**	**B**	**C**	**1**	**2**	**3**	**4**	**5**

(6)

(e) Explain the purpose of the algorithm in **Figure 2**.

(1)

(f) After a cat show, CTC needs to return the cats to their owners. They can have all the cats in the van at the same time because the show is now finished.

CTC likes to plan the return journey so that the shortest possible distance is travelled by the van. This is an example of an intractable problem.

What is meant by an intractable problem?

(2)

(g) What approach might a programmer take if asked to solve an intractable problem?

(2)

(Total 16 marks)